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Use Letter size paper only.

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PHYS 703 Final Exam

December, 2009

1. [10 points] Consider a pure electric dipole $\vec{p}$ at a distance $l > a$ from the center of a grounded conducting sphere of radius a. The dipole moment points towards the center of the sphere. Show that the image is not simply a dipole but also a charge; find the locations and the monopole and dipole moments of the image(s).

2. [8 points] A spherical surface of radius 10m is known to have an azimuthally symmetric surface charge density $\sigma(\theta)$. There is nothing (no charges, dielectrics or conductors) inside or outside the spherical surface.

(a) If the potential inside the sphere is given by
$\Phi(x, y, z) = [120 + 16(3z^2 - r^2)]$ volts,
find (to a precision of 1% or better) the potential (in volts) at the field point
$\mathrm{P}(x, y, z) = (5\sqrt{3}, 15, 10)$m.

(b) Specify the multipoles that appear in this potential.

3. [12 points] Consider a cylinder of magnetic permeability μ, and which has a radius R which is comparable to its length L. The cylinder is magnetized with uniform magnetization $\vec{M}$ parallel to its axis, which is the z-axis. The cylinder extends from $z = 0$ to $z = L$. Find the magnetic field outside the cylinder for $z > L$.

[Note: If you cannot do this, try at least to find the field outside the cylinder along the axis. If even this fails, then at least specify the most important term in the field outside.]

4. [10 points] Consider a charge q moving with constant velocity $\vec{v}$ such that at time $t = 0$ it passes through the origin

$$\vec{x}'(t) = \vec{v}t$$

(a) Obtain the retarded time t_r.

(b) Find the potential due to the charge at a point $\vec{x}$ at time t. Recall that the retarded potential for a moving charge is given by

$$\Phi(\vec{x},t) = \frac{q}{4\pi\epsilon_0 R(1 - \vec{\beta} \cdot \hat{R})}$$

where $\vec{R}$ is the displacement from the charge to the field point and it is understood that *all* quantities on the RHS are to be evaluated at the retarded time.