PHYS 703 Test 1
September, 2011

As always, you may use any valid approach, but please explain each step carefully and fully.

1. [10 points]

   State and prove the mean value theorem for electrostatic potentials in charge-free space.

2. [10 points]

   Under what conditions is the electrostatic potential (solution to the Poisson equation) unique? State and prove a uniqueness theorem.

3. [10 points]

   An electric charge q is placed at a distance d on the positive z-axis from a grounded conducting plane located at $z = 0$. Find the induced charge density on the plane and its integral (total induced charge).

Possibly useful relations

$$\int_V d^3x \left(\phi \nabla^2 \psi + \vec{\nabla}\phi \cdot \vec{\nabla}\psi \right) = \oint_S da\, \phi \frac{\partial \psi}{\partial n}$$

$$\Phi(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int_V d^3x'\, \rho(\vec{x}')G(\vec{x}, \vec{x}') + \frac{1}{4\pi} \oint_S da' \left[G(\vec{x}, \vec{x}')\frac{\partial \Phi}{\partial n'} - \Phi(\vec{x}')\frac{\partial G(\vec{x}, \vec{x}')}{\partial n'} \right]$$