Q1 for Jackson 4.7 (thanks to Colin for the question): I have a question on Jackson 4.7 b. To determine the potential explicitly do we use Green's function, given by eq. 3.114, and then let the surface go to infinity (for all points in space)? This would make the surface integral of eq 3.126 vanish and then we only have one integral to evaluate. This is the only way I can think of solving this part, yet it seems like there should be an easier way. I feel like I should be able to take my potential from (a), perform an expansion to get to the expression in (b), yet they look very different.

Ans: When you arrive at Jackson's eq. (4.2) from the previous equation, which is actually the same as eq. (1.17), for the potential, you expand 1/distance (utilizing also the addition theorem) and assume that r > r'. This is part (a). For part (b) you just repeat but this time you assume that r < r' and re-calculate the potential.

In other words, you should think of eq. (4.10) as the result of eq. (4.2) without thinking about the multipole moments defined in equation (4.3). Similarly, the solution to part (b) is an expansion of the potential for the case when r < r', obtained straight from eq. (1.17) by expanding 1/distance and without defining intervening quantities such as q_{lm}.

Q2 for Jackson 4.7 (thanks to Colin for the question): Thank you! That helped a lot and I was able to get to the correct expression. So from my understanding the solution for part b, where r<r', is a solution for the potential inside the charge distribution. Am I correct in thinking this?

Ans: Well, the charge density given extends out to infinity: $\rho(\vec{x}) = \frac{1}{64\pi} r^2 e^{-r} \sin^2\theta$
So, in principle there is no well-defined region “inside” and “outside” the charge distribution.
Thus it would have been better if Jackson had used a scale for distance, such as “a” below. Then we could talk about r << a and r >> a regions:

$$\rho(\vec{x}) = \frac{1}{64\pi} \frac{r^2}{a^2} e^{-r/a} \sin^2\theta$$