PHYS 704 Final Exam

- 1. [10 points]
Find the electric and magnetic fields appropriate for radiation from

$$\Phi(\vec{x},t) = \frac{1}{4\pi\epsilon_0 r}\left\{ Q(t_0) + \frac{\hat{r}\cdot\dot{\vec{p}}(t_0)}{c} + \frac{\hat{r}\cdot\vec{p}(t_0)}{r} \right\}$$

and

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi r}\dot{\vec{p}}(t_0)$$

by working out explicitly any derivatives you might encounter.

- 2. [10 points]
Three charges are located along the z axis, a charge $+2q$ at the origin, and charges $-q$ at $z = \pm a\cos(\omega t)$. Determine the lowest non-vanishing multipole moments, the angular distribution of radiation, and the total power radiated. Assume that $ka \ll 1$.

- 3. [10 points]
(a) Derive the parallel-velocity addition law for two successive Lorentz transformations in the same direction.
(b) Derive a formula for the Doppler effect when the velocity of the source in the observer rest frame makes an angle θ with the photon direction.

- 4. [10 points]
Consider a charged particle of charge ze traveling along the x axis with speed v.
(a) By calculating the electromagnetic energy flow through a cylinder of radius a around the path of the moving charge show that the energy lost at impact parameters b greater than a minimum value a is given by

$$\left(\frac{dE}{dx}\right)_{b>a} = -ca\,\Re\int_0^\infty d\omega\, B_3^*(\omega)E_1(\omega)$$

(b) The fields for $|\lambda a| \gg 1$ are given by

$$E_1(\omega,b) \to i\frac{ze\omega}{c^2}\left[1 - \frac{1}{\beta^2\epsilon(\omega)}\right]\frac{e^{-\lambda b}}{\sqrt{\lambda b}},$$

$$E_2(\omega,b) \to \frac{ze}{v\epsilon(\omega)}\sqrt{\frac{\lambda}{b}}e^{-\lambda b},$$

$$B_3(\omega,b) \to \beta\epsilon(\omega)E_2(\omega,b).$$

Under what conditions does Cherenkov radiation result? Define the Cherenkov angle θ_C and determine an expression for $\cos(\theta_C)$. How does the number of emitted photons depend on θ_C and on the frequency?

- 5. [10 points]
  The Larmor formula for the power radiated by an accelerating charge at non-relativistic (NR) velocities is

$$P = \frac{2}{3}\frac{q^2}{c^3}a^2 \tag{1}$$

  where the magnitude of the charge is q and the acceleration is a.
  (a) Write down a Lorentz-invariant expression which may be the proper relativistic generalization of the Larmor formula: it should reduce to the above in the NR limit and not contain anything higher than first-order in velocity derivatives. [Hint: the simplest generalization works.]
  (b) Show that your expression can be written as

$$P = \frac{2}{3}\frac{q^2}{c}\gamma^6\left[(\dot{\vec{\beta}})^2 - (\vec{\beta}\times\dot{\vec{\beta}})^2\right] \tag{2}$$

  where dots represent differentiation with respect to time t, not with respect to τ.
  (c) Consider 100 GeV electrons in a linear accelerator or in a circular accelerator. Recognize that the maximum acceleration gradient that can be provided is roughly 50 MeV/m. Use the above expression to find the radiative power loss per meter in both cases and comment on which may be higher.