PHYS 704 - Final Exam

Note: If you do not recall important formulas (and you should) just ask!

- 1. [10 points]
  (a) Starting from the Maxwell equations, obtain the wave equation for an electromagnetic wave in a uniform medium and then the Helmholtz equation for a monochromatic electromagnetic wave in such a medium.
  (b) How is this equation modified in the presence of an Ohmic conductor of conductivity σ.
  (c) A plane-polarized electromagnetic wave of frequency ω in free space is incident normally on the flat surface of a nonpermeable medium of conductivity σ and dielectric constant ϵ. Calculate the amplitude and phase of the reflected wave relative to the incident wave for arbitrary σ and ϵ.

- 2. [10 points]
  (a) Consider a single point charge q rotating in a circle of radius R at a fixed non-relativistic speed v. What is the total radiated power?
  (b) Consider a dipole consisting of two charges $\pm q$ at the two ends of an insulating rod of length $2R$ which rotates around its center at fixed angular speed such that the charges are moving at a non-relativistic speed v. Decompose this situation into oscillating dipoles and determine the angular distribution and total radiated power. Compare this result to the result of part (a).

- 3. [10 points]
  (a) A covariant formulation of gauge freedom is that the 4-gradient of any scalar function (say χ) of spacetime can be added to the 4-vector potential. Show that the electromagnetic field tensor is unaffected by such a change.
  (b) Obtain a relativstically correct "Lorentz force law" for $d\vec{p}/dt$ from the covariant equation

$$\frac{dp^\mu}{d\tau} = \frac{q}{m} F^{\mu\nu} p_\nu \tag{1}$$

  and use it to find the radius R of the circle in which a particle of charge q and momentum $p \equiv |\vec{p}| = m\gamma v$ gyrates in a uniform magnetic field $\vec{B}$ perpendicular to its motion. There is no electric field.

- 4. [10 points]
  The covariant Lagrangian for a charged particle in an electromagnetic field can be written as

$$L = -m\sqrt{U_\mu U^\mu} - J_\mu A^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{2}$$

  Use the Euler-Lagrange equations to deduce the free particle equation of motion as well as two of the Maxwell equations. Where are the other two equations in the covariant formulation? [Note that you may ignore non-essential numeric factors and factors of c.]

- 5. [10 points]
  A heavy charged particle (such as a proton) traverses matter. Obtain an expression for the average energy lost per unit distance, i.e., for dE/dx. Your result should exhibit at least an initial fall with energy followed by a relativistic rise.