Power Loss of Synchrontron Radiation

Tongtong Cao

Synchrontron Radiation

- A **synchrotron** is a particular type of cyclic particle accelerator in which the magnetic field (to turn the particles so they circulate) and the electric field (to accelerate the particles) are carefully synchronised with the travelling particle beam. In the synchrontron, charged particles are accelerated to very high speeds, and emit synchrontron radiation.

Larmor's equation

- The classical formula called Larmor’s equation for the radiated power from an accelerated electron is

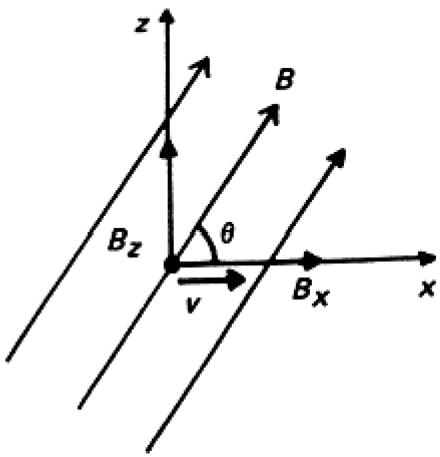
- $$P = \frac{2Ke^2}{3c^3}\left|\dot{v}\right|^2 = \frac{e^2}{6\pi\varepsilon_0 c^3}\left|\dot{v}\right|^2$$

- Larmor's equation describes the electromagnetic radiation from an accelerated charge are correct only in inertial frames where the electron velocity *v*<< *c*, but the results can be transformed to any other inertial frame by the Lorentz transformation. In this way, it is possible to calculate the total power radiated by an ultrarelativistic electron in a magnetic field. We use primed coordinates to describe an inertial frame in which the electron is temporarily at rest. Then

  Larmor's equation correctly gives

$$P' = \frac{e^2}{6\pi\varepsilon_0 c^3}\left|\dot{v}'_\perp\right|^2$$

Power in the rest frame

- 

In the rest frame,

$$f' = m_e \dot{v}' = e(E' + v' \times B') = eE'$$

$$E'_x = E_x \qquad E'_x = 0$$

$$E'_y = \gamma(E_y - vB_z) \quad \text{and so} \quad E'_y = -v\gamma B_z = -v\gamma B \sin\theta$$

$$E'_z = \gamma(E_z + vB_y) \qquad E'_z = 0$$

$$P = -\frac{dE}{dt} = -\frac{dE}{dE'} \cdot \frac{dE'}{d\tau} \cdot \frac{d\tau}{dt} = \gamma P' \frac{1}{\gamma} = P'$$

- Therefore, $$\dot{v}' = -\frac{e\gamma v B \sin\theta}{m_e}$$
- Consequently, in the rest frame, the power is

$$\mathrm{p}' = \frac{e^2|\dot{v}'|^2}{6\pi\varepsilon_0 c^3} = \frac{e^4\gamma^2 B^2 v^2 \sin^2\theta}{6\pi\varepsilon_0 c^3 m_e^2}$$

- Since P is a Lorentz invariant,

$$\mathrm{p} = \mathrm{p}' = 2\left(\frac{e^4}{6\pi\varepsilon_0^2 c^4 m_e^2}\right)\left(\frac{v}{c}\right)^2 c\,\frac{B^2}{2\mu_0}\gamma^2 \sin^2\theta = 2\sigma_T c U_{mag}\left(\frac{v}{c}\right)^2 \gamma^2 \sin^2\theta$$

- Where $\sigma_T = \dfrac{e^4}{6\pi\varepsilon_0^2 c^4 m_e^2}$ is Thomson cross-section,
- $U_{mag} = B^2/2\mu_0$ is the energy density of the magnetic field,
- θ is called the pitch angle.

Average synchrotron power

- The power formula we have obtained apply for electrons of a specific pitch angle. Particles of a particular energy, or Lorentz factor, are often expected to have an isotropic distribution of pitch angles and therefore we can work out theire average power by averaging over such a distribution of pitch angles.

$$\langle P \rangle = 2\sigma_{\mathrm{T}}\beta^2\gamma^2 c\, U_{\mathrm{B}} \langle \sin^2\theta \rangle \ .$$

$$\langle \sin^2\theta \rangle \equiv \int \sin^2\alpha d\Omega \Big/ \int d\Omega = \frac{1}{4\pi}\int \sin^2\theta \, d\Omega = \frac{2}{3}$$

$$\boxed{\langle P \rangle = \frac{4}{3}\sigma_{\mathrm{T}}\beta^2\gamma^2 c U_{\mathrm{B}}}$$

Compare to Jackson (14.31)

- Jackson (14.31) is

$$P = \frac{2e^2c}{3\rho^2}\beta^4\gamma^4 \quad \text{(Jackson ignored } 4\pi\varepsilon_0 \text{ and didn't consider average power)}$$

$$evB = \gamma m v w \Rightarrow w = \frac{eB}{\gamma m} = \frac{c\beta}{\rho} \Rightarrow B = \frac{\gamma\beta m c}{e\rho}$$

$$\frac{<P> \cdot 4\pi\varepsilon_0}{2/3} = \frac{\frac{4}{3}\sigma_T \beta^2 c U_B \gamma^2 \cdot 4\pi\varepsilon_0}{2/3} = \frac{\frac{4}{3}\left(\frac{e^4}{6\pi\varepsilon_0^2 m^2 c^4}\right)\beta^2 c \frac{\left(\frac{\gamma\beta m c}{e\rho}\right)^2}{2\mu_0}\gamma^2 \cdot 4\pi\varepsilon_0}{2/3} = \frac{2e^2c}{3\rho^2}\beta^4\gamma^4$$

- Power is the fourth power of the particle energy E. The mass dependence means that this rate of energy loss is about 10^{13} times larger for electrons than for protons of the same energy.

Thank you!