PHYS 704 - Final Exam

Note: If you do not recall important formulas (and you should) just ask!

- 1. [10 points]
  A circular wire loop of radius b lies in the x-y plane with center at the origin. It carries an alternating current $I(t) = I_0 \cos(\omega t)$. Find (a) the electric and magnetic fields at distant points, (b) the differential and total power radiated, and (c) the radiation resistance of this configuration.

- 2. [10 points]
  A perfectly conducting flat screen occupies half of the x-y plane (i.e., $x < 0$). A plane wave of intensity I_0 and wave number k is incident along the z axis from the region $z < 0$. Consider the diffracted fields in the plane parallel to the x-y plane defined by $z = Z > 0$. Let the coordinates of the observation point be $(X, 0, Z)$. In the usual scalar Kirchoff approximation and in the limit $Z \gg X$ and $\sqrt{kZ} \gg 1$ find the diffracted field $\psi(X, 0, Z, t)$ in terms of $\xi = (k/2Z)^{1/2} X$. An integral is fine.

- 3. [10 points]
  (a) A wire is uncharged in the O-frame and carries a current $I = 10$ Amps which results from a line of positive charges moving to the right (along the x-axis) with speed $v = 0.8c$ and a line of negative charges at rest. What is the net electric charge density of the wire (in C/m) in the O'-frame which is moving along the x-axis with velocity $v = 0.8c$. What is the current (in Amps) carried by the wire in the O'-frame?
  (b) The *rapidity* η of a particle can be defined as $\tanh^{-1} \beta$. Show that an interval in rapidity is invariant under longitudinal Lorentz boosts.

- 4. The BMT equation for the time dependence of the spin of a charged particle is

$$\frac{dS^\alpha}{d\tau} = \frac{e}{mc}\left[\frac{g}{2} F^{\alpha\beta} S_\beta + \frac{1}{c^2}\left(\frac{g-2}{2}\right) v^\alpha \left(S_\lambda F^{\lambda\mu} v_\mu\right)\right]$$

  (a) What is the corresponding equation for the 4-momentum?
  (b) How does the spin of a muon with $g = 2$ precess around its momentum?
  (c) Explain qualitatively what is different when g is slightly larger than 2 (the real case).
  (d) Find an expression for the time-component of the spin 4-vector in terms of the spin and velocity 3-vectors.

- 5. [10 points]
  (a) Find an approximate expression for the maximum energy transferred to an electron at rest by a charged particle that moves past the electron at a relativistic speed. What does this expression give for a LHC proton of energy 3.5 TeV?
  (b) Use the above expression to find, at least approximately, the energy loss per unit distance by a charged particle traversing a medium. Give reasons for the limits of any integral you may evaluate.