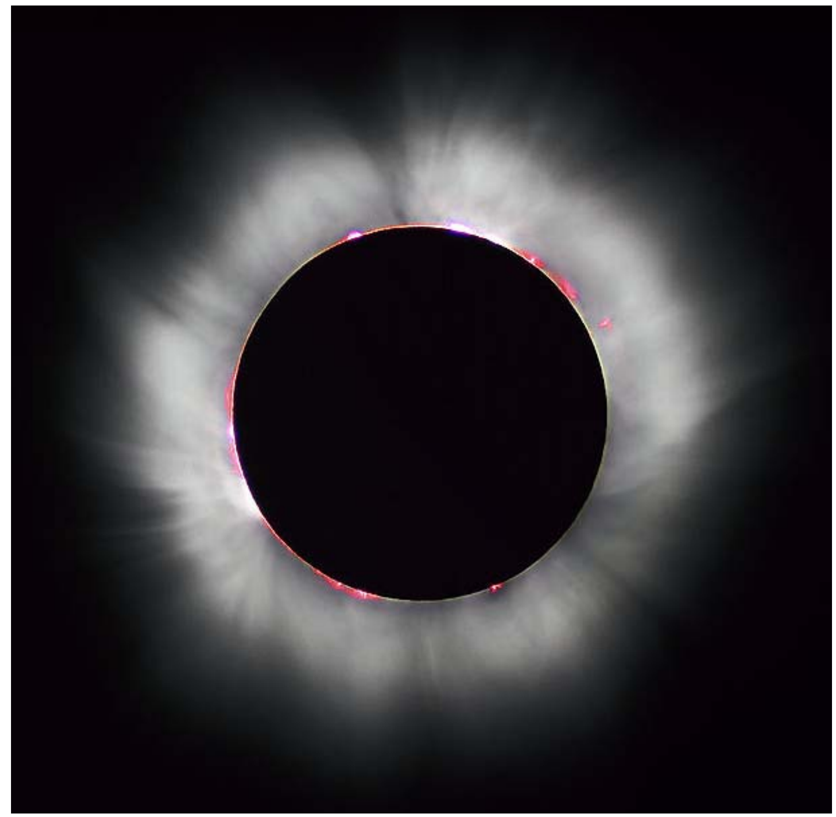
Magnetic Reconnection and the Aurora

Outline

- Introduction
- Frozen-in Field Lines
- Field Line Topology
- Non-Ideal Effects/Magnetic Reconnection
- Conclusion

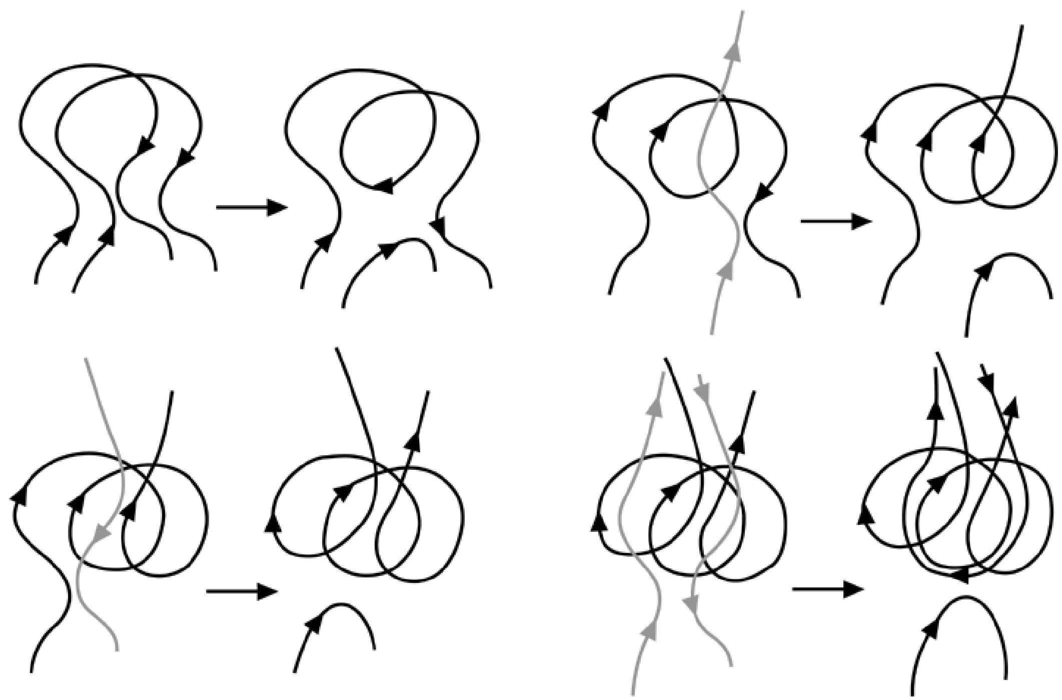
Interplanetary Plasma

- Corona of the sun
- Mostly Ionized Hydrogen
- Carries “Frozen-in” Magnetic Field

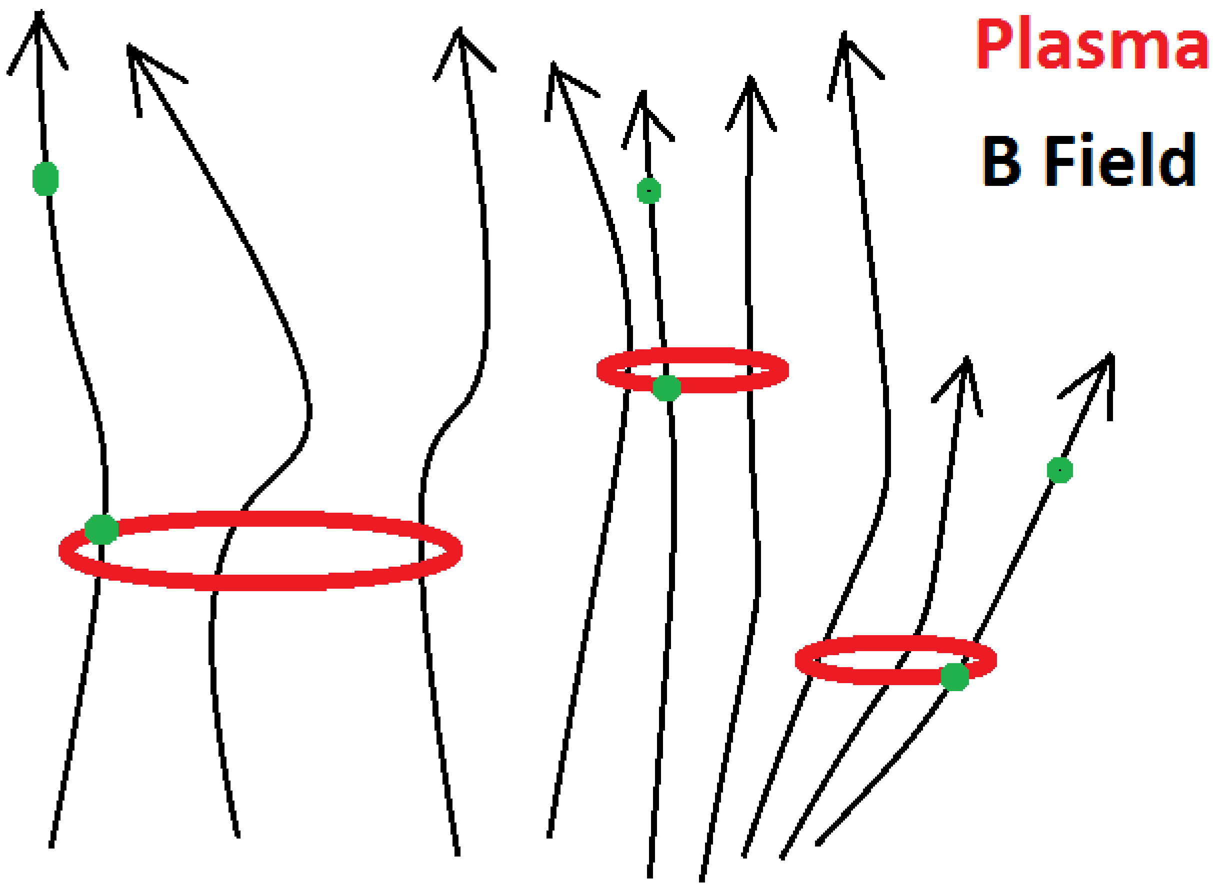


Magnetic Reconnection

- Auroral Trigger Mechanism
- Release of stored Magnetic Energy
- Conversion to Kinetic Energy
- Return to a Relaxed State



Frozen-In Fields



Magnetic Flux Tube Conservation

Generalized Ohm's Law (c=1) $\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \boldsymbol{R}$

Ideal Approximation $\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \boldsymbol{0}$

Faraday's Law $\partial_t \mathbf{B} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B})$

Magnetic Flux $\Phi = \int_S \boldsymbol{B} \cdot d\boldsymbol{S}$

Change in Flux $\frac{d\Phi}{dt} = \int_S \partial_t \boldsymbol{B} \cdot d\boldsymbol{S} + \oint_{\partial S} \boldsymbol{B} \cdot (\boldsymbol{v} \times d\boldsymbol{l})$

but $$\int_S \partial_t \boldsymbol{B} \cdot d\mathrm{S} = \oint_{\partial S} (\boldsymbol{v} \times \boldsymbol{B}) \cdot d\boldsymbol{l} = -\oint_{\partial S} \boldsymbol{B} \cdot (\boldsymbol{v} \times d\boldsymbol{l})$$

$$\Rightarrow \frac{d\Phi}{dt} = 0$$

Field Lines are "Frozen-In" to the plasma.

Conservation of Magnetic Topology

Consider two generating functions for the field lines

$$\boldsymbol{F}_{\boldsymbol{B}}(\boldsymbol{x}, s, t) \quad \text{and} \quad \boldsymbol{F}_{\boldsymbol{u}}(\boldsymbol{x}, t, t_0)$$

with the following properties

$$\partial_s \boldsymbol{F}_{\boldsymbol{B}}(\boldsymbol{x}, s, t) = \boldsymbol{B}(\boldsymbol{x}, t) \; with \; \boldsymbol{F}_{\boldsymbol{B}}(\boldsymbol{x}, 0, t) = \boldsymbol{x}$$

$$\partial_t \boldsymbol{F}_{\boldsymbol{u}}(\boldsymbol{x}, t, t_0) = \boldsymbol{u}(\boldsymbol{x}, t) \; with \; \boldsymbol{F}_{\boldsymbol{B}}(\boldsymbol{x}, t_0, t_0) = \boldsymbol{x}$$

Let a field line connect two points **x** and **y** at time t_0

$$\boldsymbol{y} = \boldsymbol{F}_{\boldsymbol{B}}(\boldsymbol{x}, s, t_0) \text{ for } some\ s$$

and let **x'** and **y'** be the points to which **x** and **y**

are transported after a time $t\text{-}t_0$.

$$\boldsymbol{x}' = \boldsymbol{F}_{\boldsymbol{u}}(\boldsymbol{x}, t, t_0)$$

$$\boldsymbol{y}' = \boldsymbol{F}_{\boldsymbol{u}}(\boldsymbol{y}, t, t_0) = \boldsymbol{F}_{\boldsymbol{u}}(\boldsymbol{F}_{\boldsymbol{B}}(\boldsymbol{x}, s, t_0), t, t_0)$$

Magnetic Topology cont'd

The topology of the field lines will be considered conserved if **x'** and **y'** still lie along the same field line, i.e.

$$y' = F_B(x', s', t) = F_B\left(F_u\left(x, t, t_0\right), s', t\right)$$

$$\Rightarrow F_u\left(F_B(x', s, t), t, t_0\right) = F_B\left(F_u\left(x, t, t_0\right), s', t\right)$$

Taking the second mixed derivative at s=0 and $t=t_0$ gives

$$\partial_s \partial_t F_u\left(F_B\left(x, s, t_0\right), t, t_0\right) = \partial_t \partial_s F_B\left(F_u\left(x, t, t_0\right), s', t\right)$$

$$= \partial_s\left[u\left(F_B\left(x, s, t_0\right), t_0\right)\right]_{s=0} = \partial_t\left[B\left(F_u\left(x, t, t_0\right), t\right) \cdot \partial_s s'\right]_{t=t_0}$$

$$= \nabla u \cdot B = \partial_t B + B \cdot \nabla u + B \partial_t \partial_s s'$$

Consequence

The total time derivative of B
is proportional to B.

$$\frac{d\boldsymbol{B}}{dt} = \boldsymbol{B}\sigma + \boldsymbol{B} \cdot \nabla \boldsymbol{u}$$

Magnetic nulls cannot be created, only moved.
Non ideal effects give rise to new
magnetic nulls, i.e. magnetic reconnection.