PHYS 704 - Final Exam

- 1. [10 points]
  A plane electromagnetic wave with electric field of amplitude E_0 and angle of incidence θ comes from a medium of refractive index n on to an interface with another medium of refractive index n'. The reflected wave has amplitude E_0' and makes an angle θ' with the normal to the interface; the transmitted wave has amplitude E_0'' and makes an angle θ'' with the normal.
  (a) State the boundary conditions on the fields at the interface and use them to derive relations between the amplitudes.
  (b) Derive thereby the ratios of reflected and transmitted amplitudes to the incident amplitude and the transmission coefficient T.

- 2. [10 points]
  Two halves of a spherical metallic shell of radius R and infinite conductivity are separated by a very small insulating gap. An alternating potential is applied between the two halves of the sphere so that the potentials are $\pm V \sin \omega t$. In the long-wavelength limit, find the radiation fields, the angular distribution of radiated power, and the total radiated power from the sphere. [Take the axis of symmetry to be the z-axis and the origin to be the center of the sphere.]

- 3. [10 points]
  (a) The theory of waveguides refers to certain frequencies ω_{mn}. What do these mean? The possibilities are that they are resonant frequencies, or lower cut-off frequencies (higher frequencies can propagate), or upper cut-off frequencies (lower frequencies can propagate). Explain your answer.
  (b) An atom in the atmosphere is excited by sunlight and re-radiates in response. Why do you expect to see mainly blue light due to this scattering of sunlight? In which direction is the light coming towards you polarized? [Be mindful of three dimensions, consider also the direction of the incident sunlight, and explain your answer very clearly; unclear answers will receive zero points.]
  (c) Write down a covariant Lagrangian and action for a free particle and thereby derive its equation of motion.

- 4. [10 points]
  In this kinematics problem we consider a neutrino to be massless and "high energy" to mean energies much greater than the rest energy of a proton, which has mass M. Except for a particle at rest, assume all energies are high in both the lab and the c.m. frame.

  A high energy neutrino of energy E and 4-momentum p is incident on a proton at rest in the lab. The proton's 4-momentum is P.
  (a) What is the c.m. energy of the collision?
  (b) The collision produces an electron of energy E' (4-momentum p') and a large number of other particles with total energy E_h and 4-momentum p_h. The ratio $y \equiv E_h/E$ is known as the "inelasticity" and can be considered as the ratio of two Lorentz-invariants; use dot products of available 4-momenta to express it as such.
  (c) Relate y (or $1 - y$) to the c.m. scattering angle of the lepton θ^*, i.e., the angle between the emerging electron momentum and the incident neutrino momentum in the c.m. frame.

- 5. [10 points]
  (a) Work out the components of $F^{\mu\nu}$ in terms of components of the familiar $\vec{E}$ and $\vec{B}$ fields. Use either SI units or Gaussian units and state which units you are using.
  (b) Starting from the relativistic equation for $dp^\mu/d\tau$ obtain the familiar Lorentz Force law in 3-vector form.
  (c) If the non-relativistic form of the momentum is $m\vec{v}$, what is the relativistically correct expression?
  (d) A charged particle travels along a circular path in a uniform magnetic field of strength B_0. Find a relativistically correct expression for the radius R of the circle in terms of the particle's charge q, and momentum $p = |\vec{p}|$ using the Lorentz Force law.