Presentation / Physics 706 Spring 2013

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According to our course, we have seen that when light propagate into a dielectric, the index n is such as:

$$n^2 = \frac{\epsilon}{\epsilon_0} = 1 + N\gamma_{mol} = 1 + \frac{Ne^2}{m\epsilon_0}\sum_j \frac{f_j}{\omega_j^2 - \omega^2 - i\gamma_j\omega}$$

with f_j= oscillator strength

$p = \gamma_{mol}\epsilon_0 E$

N = number of electrons / unit volume

$\sum_j f_j = Z$ (number of electrons / molecules)

ω_j = resonant or binding frequencies

For γ_j negligible (we do not look for absorption),

$$n \approx 1 + \frac{Ne^2}{2m\epsilon_0}\sum_j \frac{f_j}{\omega_j^2 - \omega^2} \quad (\text{ for } \quad \omega \neq \omega_j \ )$$

For radiation such as $\omega < \omega_j$ (typically, the nearest significant resonances lie in UV for transparent substances):

$$\frac{1}{\omega_j^2 - \omega^2} \approx \frac{1}{\omega_j^2}\ \frac{1}{1 - \frac{\omega^2}{\omega_j^2}} \approx \frac{1}{\omega_j^2}\left(1 + \frac{\omega^2}{\omega_j^2}\right) \approx \frac{1}{\omega_j^2} + \frac{\omega^2}{\omega_j^4}$$

$$\Rightarrow \ n = 1 + \frac{Ne^2}{2m\epsilon_0}\sum_j \frac{f_j}{\omega_j^2} + \omega^2\frac{Ne^2}{2m\epsilon_0}\sum_j \frac{f_j}{\omega_j^4}$$

Using the expression for the wavelength of light in vacuum $\lambda = \frac{2\pi c}{\omega}$ we get:

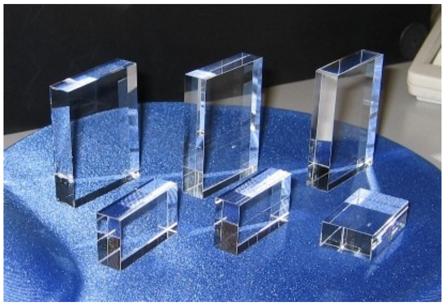
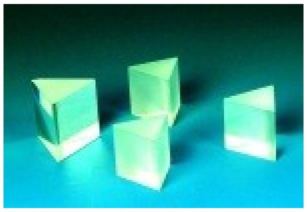
$$n = 1 + A\left(1 + \frac{B}{\lambda^2}\right)$$ which is called the “Cauchy formula”

A is the coefficient of refraction
B is the coefficient of dispersion

ω_j account for refraction index and its frequency dependance. Ordinarly, n increases with ω .

Examples of coefficient A and B :

	A	AB (μm^2)
Fused silica	0.458	0.00354
Dense flint glass SF10	0.728	0.01342

Fused silica	Dense flint
 Retrived from *http://www.cristaltechno.com/FS_en.htm*	 Retrived from *http://www.science2education.co.uk/product/PH0583C*

Application: SC manufacturing

The Cauchy's formula can be used to determine the thickness of a thin layer. In SC manufacturing, the Si O_2 layer for a transistor may be between 5.0 and 10.0 nm, and an uncertainty of a few Angstrom is required.
A beam is sent onto a sample (Si O_2 layer on a Silicon substrate), and after interference due to the thin layer, the light is analyzed by a spectrometer. The dispersion term of the index is needed to compare the measured spectra to the known index.
Thickness, roughness, uniformity are determined thanks to this technique.

References and interesting articles:

- Lecture Phys 706 Dr. Purohit
- Griffiths "Introduction to electrodynamics"
- Jackson "Classical Electrodynamics" 3rd edition
- http://en.wikipedia.org/wiki/Cauchy's_equation *(Examples of coefficient A and B)*

http://www.semiconsoft.com/html/welcome.htm *(application to the industry: thin film measurement system, with a library of more than 500 material)*

http://www.chem.agilent.com/Library/applications/uv90.pdf *(thickness measurement using reflectance spectroscopy)*

http://www.ub.edu/optmat/spie.pdf *(interesting example of characterization for a material)*