Resolving Power of Diffraction Gratings

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Diffraction gratings are used in many fields in physics, especially astronomy. There are many equations that govern the physics of these gratings, including the resolving power equation. The resolving power is utilized to through dispersion to give the require resolution for spectral analysis of light sources.

1 Introduction

Diffraction gratings are used in many fields in physics, especially astronomy. In astronomy, diffraction gratings are used in spectrographs to analyze the light of astronomical objects that is gathered by telescopes. These gratings use the principles of constructive and destructive interference of light to spread the different wavelengths out based on the wave nature of light and the diffraction grating equation, Eq. 1, where d is the grating spacing, θ is the angle of the incoming light, n is diffraction order, and λ is the wavelength of light.

$$d \sin\theta = n\lambda \tag{1}$$

The path length of light is related to wavelength of light and the structure of grating. This allows for the analysis of the elemental makeup of the objects in question based on the resulting spectra. The special resolution of the resulting spectra is determined by there resolving power equation.

1.1 Types of Diffraction Gratings

There are two different types of diffraction gratings that both utilize the properties of electromagnetic wave interfaces at a dielectric medium. These are reflection (Fig. 1a) and transmission gratings (Fig. 1b).

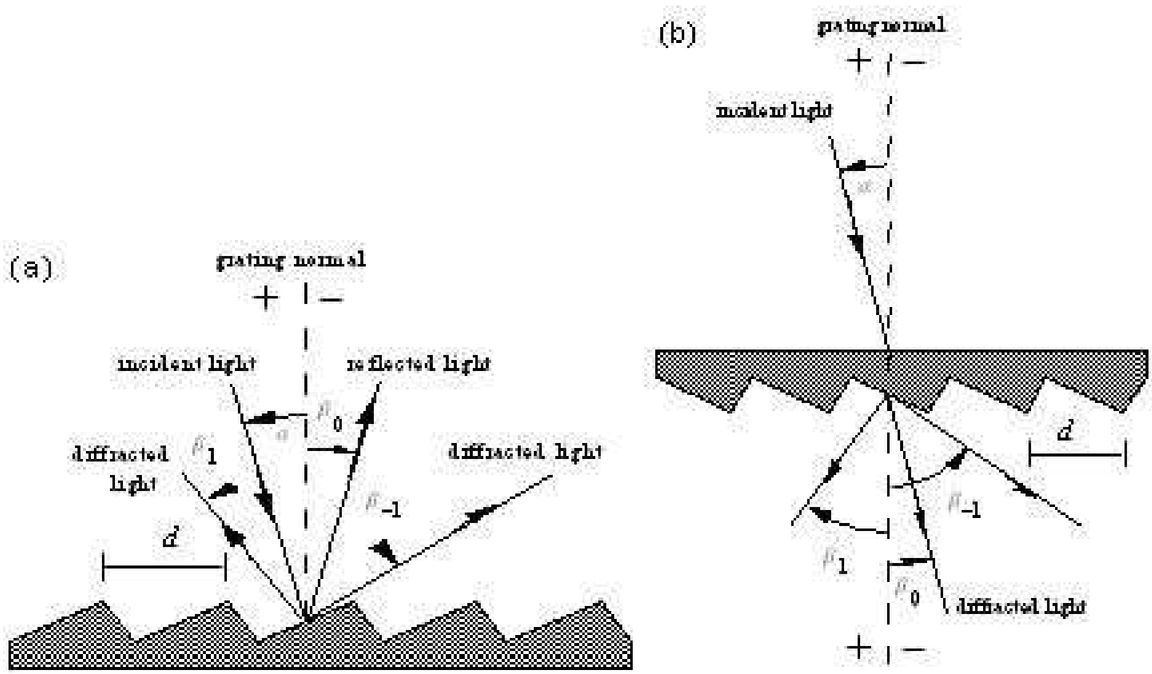

Figure 1: Two Types of Diffraction Grating with $\theta = \alpha + \beta$: (a) Reflection Gratings (b) Transmission Grating[1]

2 The Resolving Power Equation

The resolving power is a dimensionless quantity of the average wavelength λ divided by the limit of resolution, or the difference in wavelength between two lines of equal intensity that can be distinguished, $\mathrm{d}\lambda$.

The resolving power R, is dependent on the diffraction order n, and the total number of grooves that are illuminated on the surface of the grating by the light source N as shown in Eq. 2 [3].

$$R = \frac{\lambda}{\mathrm{d}\lambda} = nN \tag{2}$$

2.1 Derivation

The fourier coefficients that correspond to the amplitude of the n^{th} order of the grating, where $g(x)$ is the grating transmittance (or reflectance), is given by Eq. 3[2], when an infinitely extend diffraction grating is assumed.

$$G_n = \frac{1}{d}\int_{-\frac{d}{2}}^{+\frac{d}{2}} g(x) e^{in\frac{2\pi x}{d}}\,\mathrm{d}x \tag{3}$$

When the change of variables, $\theta = n2\pi x/d$, is applied, and the integral evaluated, the result is Eq. 4[2].

$$G_n = \frac{1}{2n\pi}\int_0^{na2\pi} e^{i\theta}\,\mathrm{d}\theta = \frac{1}{2n\pi}(1 - e^{ina2\pi}) \tag{4}$$

The magnitude of Eq. 4[2] is squared, is the intensity of the diffractions, as shown in Eq. 5[2].

$$I_n = |G_n|^2 = \frac{\sin^2(n\pi\theta)}{n^2\pi^2} \tag{5}$$

The intensity, as shown in Eq. 5[2], peaks at a phase difference between two interacting wavelets of [3]:

$$\delta = 2n\pi \tag{6}$$

This phase difference is also equal to [3]:

$$\delta = \frac{2\pi}{\lambda} d \sin\theta \tag{7}$$

The peak phase difference is given by Eq. 8 [3].

$$\mathrm{d}\delta = \frac{2\pi}{N} \tag{8}$$

The grating equation, Eq. 1, when differentiated gives Eq. 9 [3].

$$d\cos\theta\,\mathrm{d}\theta = n\,\mathrm{d}\lambda \tag{9}$$

This equation is then substituted into Eq. 10, which is the differentiated form of Eq. 7 [3].

$$\mathrm{d}\delta = \frac{2\pi}{\lambda} d\cos\theta\,\mathrm{d}\theta \tag{10}$$

That equation is then set equal to Eq. 8 to give Eq. 11 [3].

$$\frac{2\pi}{N} = \frac{2\pi}{\lambda} n\,\mathrm{d}\lambda \tag{11}$$

When Eq. 12 is inverted, Eq. 2, the resolving power equation, is recovered [3].

$$\frac{1}{Nn} = \frac{\mathrm{d}\lambda}{\lambda} \tag{12}$$

3 Significance of the Resolving Power Equation

As discussed above the resolving power is directly dependent on the diffraction order of interest and the number of groves illuminated by the source. The resolving power gives an expression of if two spectral lines with be resolved. This distance between the two lines is a minimum when the peak of one line is at the minimum of the second line. If two lines are father apart then this minimum resolved distance then they will be resolved. This can be equated to an angular dispersion through the total aperture of the grating.

3.1 Angular Dispersion

This angular dispersion is characterized by the dispersive power equation shown in Eq. 13. The resolving power is equal to the total aperture of the grating times this dispersive power[4].

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = \frac{n}{(d)\cos(\theta)} \tag{13}$$

This shows that the dispersion of the light is based on the wavelength λ, the order of diffraction n, and the grating spacing d. In addition that means that the resolving power is dependent on the wavelength, which is also shown in the resolving power equation itself.

3.2 Spectral Analysis

The resolving power of diffraction gratings are important in spectral analysis. This dispersion based on order and wavelength is important to spectral analysis because it shows that at higher order light will be spread out farther giving higher spectral resolution. In astronomy and other fields, this higher spectral resolution is utilized to give the resolution to examine the spectral structure. This analysis allows for compositional analysis of the object that is producing the light through absorption and emission. The higher resolution allows for determination of fine and hyper fine structure and the chemical make up.

References

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