NonRelativistic Quantum Mechanics Formulas

$$|x_\pm\rangle = \frac{|+\rangle \pm |-\rangle}{\sqrt{2}} \qquad |y_\pm\rangle = \frac{|+\rangle \pm i|-\rangle}{\sqrt{2}} \qquad \text{Norm: } \sqrt{\langle\alpha|\alpha\rangle} \qquad \text{Completeness: } \sum_i |i\rangle\langle i| = 1$$

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geq \frac{1}{4}|\langle[A,B]\rangle|^2$$

Change of Basis: If $\quad |b^{(i)}\rangle = U|a^{(i)}\rangle \quad$ where $\quad U = \sum_i |b^{(i)}\rangle\langle a^{(i)}| \quad$ then

Column vectors: (New) = $(U^\dagger)$(Old) $\qquad$ Operators: $X' = U^\dagger X U$

$$[x_i, x_j] = 0, \qquad [x_i, p_j] = i\hbar\delta_{ij} \qquad [p_i, p_j] = 0 \qquad \langle\vec{x}|\vec{p}\rangle = \frac{1}{(2\pi\hbar)^{3/2}} \exp\left(\frac{i\vec{p}\cdot\vec{x}}{\hbar}\right)$$

$$U(t, t_0) = T\left[\exp\left(-\left(\frac{i}{\hbar}\right)\int_{t_0}^{t} dt'\, H(t')\right)\right]$$

$$A^{(H)}(t) = \mathcal{U}^\dagger(t) A^{(S)} \mathcal{U}(t) \qquad \frac{dA^{(H)}}{dt} = \frac{i}{\hbar}[H, A^{(H)}]$$

$$a = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i\hat{p}}{m\omega}\right) \qquad [a, a^\dagger] = 1 \qquad \int_{x_1}^{x_2} dx\, \sqrt{2m(E - V(x))} = (n + \frac{1}{2})\pi\hbar$$

NonRelativistic Quantum Mechanics Formulas, contd.

$$\int_{-1}^{1} P_m(x)P_n(x)dx = \frac{2\delta_{mn}}{2n+1}$$

$[J_z, T_q^{(k)}] = \hbar q T_q^{(k)}$

$[J_\pm, T_q^{(k)}] = \hbar\sqrt{k(k+1) - q(q\pm 1)}\; T_{q\pm 1}^{(k)}$

$\mathcal{D}^\dagger(R) T_q^{(k)} \mathcal{D}(R) = \sum_{q'=-k}^{k} \mathcal{D}_{qq'}^{(k)*}(R) T_{q'}^{(k)}$

$[\vec{J}\cdot\hat{n}, T_q^{(k)}] = \sum_{q'} T_{q'}^{(k)} \langle k\, q'|\vec{J}\cdot\hat{n}\,|k\, q\rangle$

$T_q^{(k)} = \sum_{q_1}\sum_{q_2} \langle k_1 k_2; q_1 q_2 | k_1 k_2; kq\rangle X_{q_1}^{(k_1)} Z_{q_2}^{(k_2)}$

$\langle \alpha', j'm'|T_q^{(k)}|\alpha, jm\rangle = \langle jk; mq|jk; j'm'\rangle \frac{\langle\alpha' j'||T^{(k)}||\alpha j\rangle}{\sqrt{2j+1}}$

$a = \sqrt{\frac{m\,\omega}{2\hbar}}(x + \frac{i\,p}{m\,\omega})$

$x = \sqrt{\frac{\hbar}{2m\,\omega}}(a + a^\dagger)$

$[a, a^\dagger] = 1$

$H = \hbar\,\omega(a^\dagger a\, + \frac{1}{2})$

$a|n\rangle = \sqrt{n}|n-1\rangle$

$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$

$\Pi^\dagger\,\vec{x}\,\Pi = -\vec{x}$

$\Pi\,|\alpha, \ell\, m\rangle = (-1)^\ell|\alpha, \ell\, m\rangle$

$\Theta|\ell, m\rangle = (-1)^m|\ell, m\rangle$

$\Theta\,\vec{x}\,\Theta^{-1} = \vec{x},$

$\Theta\,\vec{p}\,\Theta^{-1} = \vec{p},$

$\Theta\,\vec{J}\,\Theta^{-1} = -\vec{J}$

$|n\rangle = |n^{(0)}\rangle + |\delta n^{(1)}\rangle + |\delta n^{(2)}\rangle + \cdots$

$\Delta E_n^{(1)} = V_{nn}$

$|\delta n^{(1)}\rangle = \sum_{k\neq n} \frac{V_{kn}}{E_n^{(0)} - E_k^{(0)}}|k^{(0)}\rangle$

$\Delta E_n^{(2)} = \sum_{k\neq n} \frac{|V_{nk}|^2}{E_n^{(0)} - E_k^{(0)}}$

$\dot{c}_n(t) = -\frac{i}{\hbar}\sum_m c_m(t) e^{i\omega_{nm}t} V_{nm}$

$c_n^{(1)}(t) = -\frac{i}{\hbar}\int_0^t V_{ni}\; e^{i\,\omega_{ni}t'} dt'$

$$c_n^{(2)}(t) = \left(-\frac{i}{\hbar}\right)^2 \sum_m \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt''\; V_{nm}(t') e^{i\omega_{nm}t'} V_{mi}(t'') e^{i\omega_{mi}t''}$$

$|\psi^{(\pm)}\rangle = |\phi\rangle + \frac{1}{E - H_o \pm i\epsilon} V \Big|\psi^{(\pm)}\rangle$

$f_\ell(k) = -\frac{\pi\, T_\ell(E)}{k}$

$f(\vec{k}', \vec{k}) = -\frac{1}{4\pi}\frac{2m}{\hbar^2}(2\pi)^3\langle\vec{k}'|T|\vec{k}\rangle$

$f(\vec{k}', \vec{k}) = \sum_{\ell=0}^{\infty}(2\ell+1) f_\ell(k) P_\ell(\cos\theta)$

$S_l(k) = e^{2i\delta_l} = 1 + 2ikf_l(k)$

$\sigma_{\rm tot} = \frac{4\pi}{k^2}\sum_l (2l+1)\sin^2\delta_l$

$f^{(1)}(\vec{k}', \vec{k}) = -\frac{m}{2\pi\hbar^2}\int d^3x'\; e^{i\vec{q}\cdot\vec{x}'}\; V(\vec{x}')$

$f^{(1)}(\theta) = -\frac{2m}{\hbar^2 q}\int dr\, r V(r)\sin(qr)$

$$A_l(r) = e^{i\delta_l}[\cos\delta_l j_l(kr) - \sin\delta_l n_l(kr)]$$

$j_0(x) = \sin(x)/x$

$n_0(x) = -\cos(x)/x$

$j_l(x) \xrightarrow[x\to\infty]{} \sin(x - l\pi/2)/x$

$n_l(x) \xrightarrow[x\to\infty]{} -\cos(x - l\pi/2)/x$